Additional problems

Questions 1-3 (no solutions given – you can check the answers by differentiation)

In these 3 problems, you will assume steady flow so that Darcy's law applies, i.e., for horizontal flow:

$$q = -K\frac{dH}{dx} = -K\frac{dh}{dx}$$

or, for vertical flow:

$$q = -K\frac{dH}{dz} = -K\left(\frac{dh}{dz} + 1\right)$$

Assume that the z axis is pointed upwards, so that $H = h + z$.

Q1. Consider Darcy's law for horizontal flow. Solve this equation for the case:

$$K = K_x(x)K_h(h)$$

with

$$h(x = 0) = h_0$$

where h_0 is constant.

Show that the solution is:

$$-q\int_0^x \frac{d\bar{x}}{K_x(\bar{x})} = \int_{h_0}^h K_h(\bar{h})d\bar{h}$$

Q2. Consider Darcy's law for vertical flow. Solve this equation for the case:

$$K = K_z(z)$$

with

$$h(z = 0) = h_0$$

where h_0 is constant.

Show that the solution is:

$$h = h_0 - \int_0^z \left[1 + \frac{q}{K_z(\bar{z})}\right] d\bar{z}$$

Q3. Repeat Q2 for the case:

$$K = K_h(h)$$

Show that the solution is:

$$z = -\int_{h_0}^h \left[1 + \frac{q}{K_h(\bar{h})}\right]^{-1} d\bar{h}$$

Q4. Consider the Horton infiltration equation. For rainfall rate p where $i_f < p < i_0$, show that the ponding time, t_p, is given by:

$$t_p = \frac{1}{\beta}\frac{i_0 - p}{p - i_f}$$

The solution is given below – try the problem yourself first.

Q4 – Solution

The Horton equation for cumulative infiltration is:

$$I = i_f t + \frac{1}{\beta}\left(i_0 - i_f\right)[1 - exp(-\beta t)]$$

so that:

$$i = \frac{dI}{dt} = i_f + \left(i_0 - i_f\right)exp(-\beta t)$$

The equation for i can be used to eliminate the exponential factor from the equation foor I, i.e., using the equation for i,

$$i_0 - i = \left(i_0 - i_f\right)[1 - exp(-\beta t)]$$

so the equation for I becomes:

$$I = i_f t + \frac{i_0 - i}{\beta}$$

For rainfall rate p, the ponding time is t_p, determined from $i(t_p) = p$. Also, at time t_p, $I = pt_p$. Then, the above equation becomes:

$$pt_p = i_f t_p + \frac{i_0 - p}{\beta}$$

Solving for t_p gives the required result.